Crystallinity of Galois representations associated to regular algebraic cuspidal automorphic representations of GL_n

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Abstract

I will discuss local-global compatibility at p for the p-adic Galois representations constructed by Harris-Lan-Taylor-Thorne and Scholze. More precisely, let $r_p(\pi)$ denote an n-dimensional p-adic representation of the absolute Galois group of a CM field F attached to a regular algebraic cuspidal automorphic representation π of $\mathrm{GL}_n(\mathbb{A}_F)$. For any prime $v \mid p$ of F such that π_v is unramified, we show that $r_p(\pi)|_{\mathrm{Gal}(\overline{F}_v/F_v)}$ is crystalline.

To prove the above, we use the fact that the representations $r_p(\pi)$ can be constructed as a sub-representation of a p-adic Galois representation associated to an overconvergent $GU(n,n)$-automorphic representation Π. We can then construct a certain one-parameter family containing Π and a Zariski-dense set of points whose associated Galois representations are already known to be crystalline. Using a result of R. Liu, we can show that each specialization within this family has n crystalline periods, and conclude the result by proving that the n crystalline periods are all distinct periods of $r_p(\pi)$. If time permits, I will also discuss how such an argument can be used to prove that the Galois representations $r_p(\pi)$ are de Rham at v when there is no unramifiedness assumption on π_v.